

# Chapter 11

## Modal Analysis

11.1 Step-by-Step: Gearbox

11.2 Step-by-Step: Two-Story Building

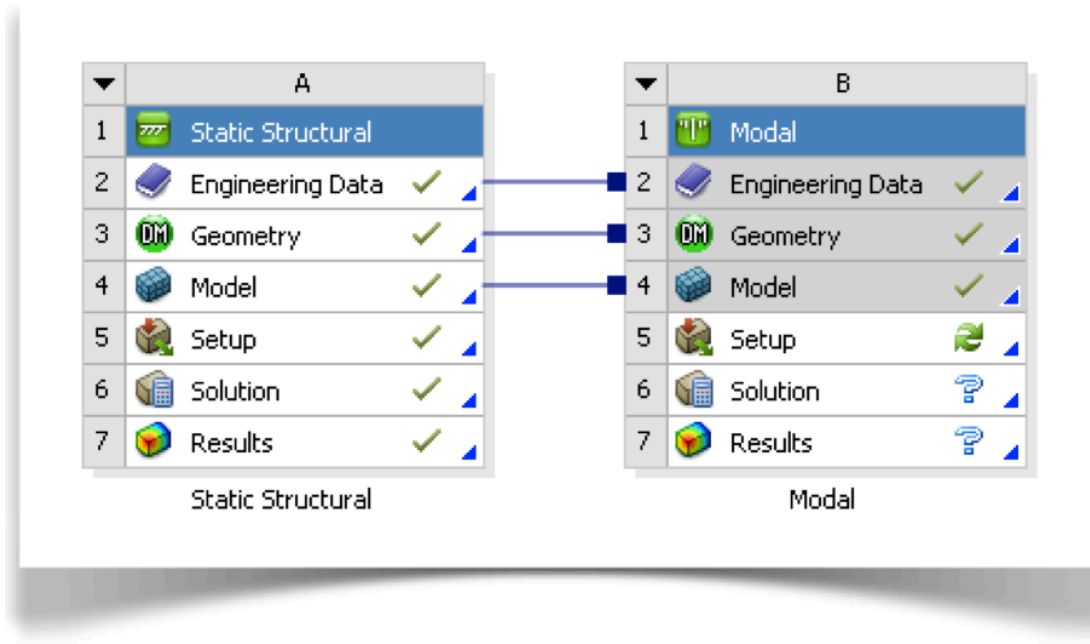
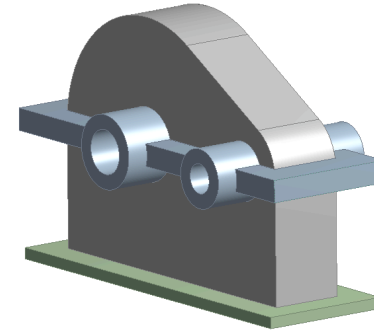
11.3 More Exercise: Compact Disk

11.4 More Exercise: Guitar String

11.5 Review

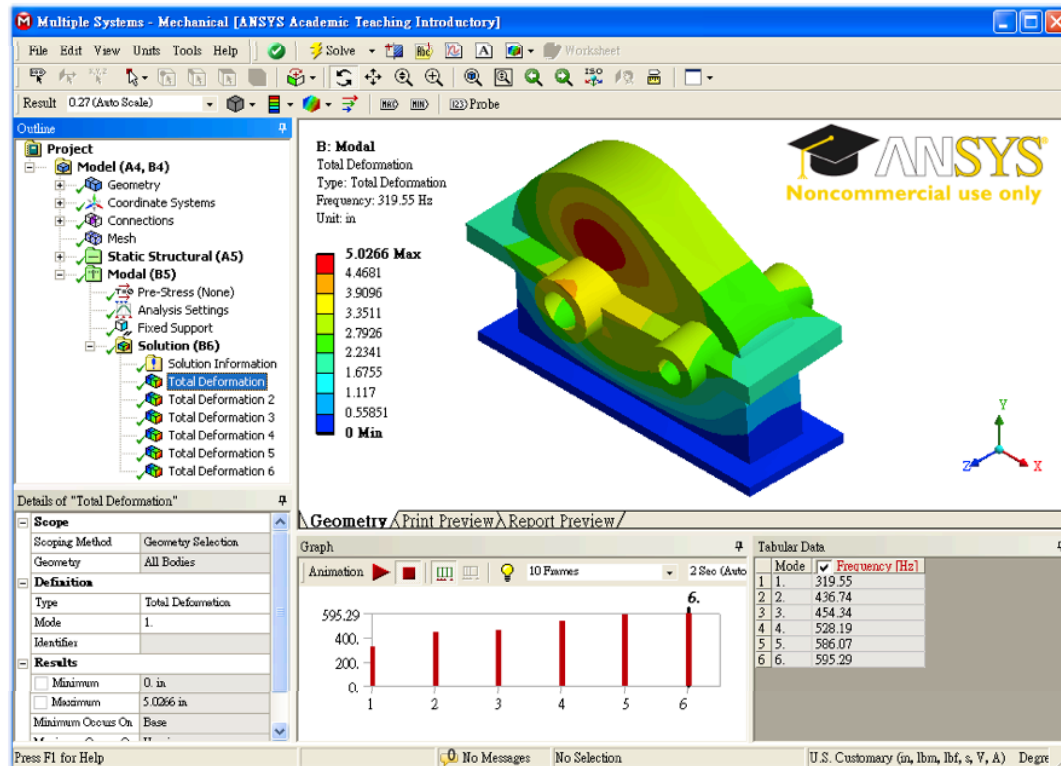
# Section 11.1

## Gearbox



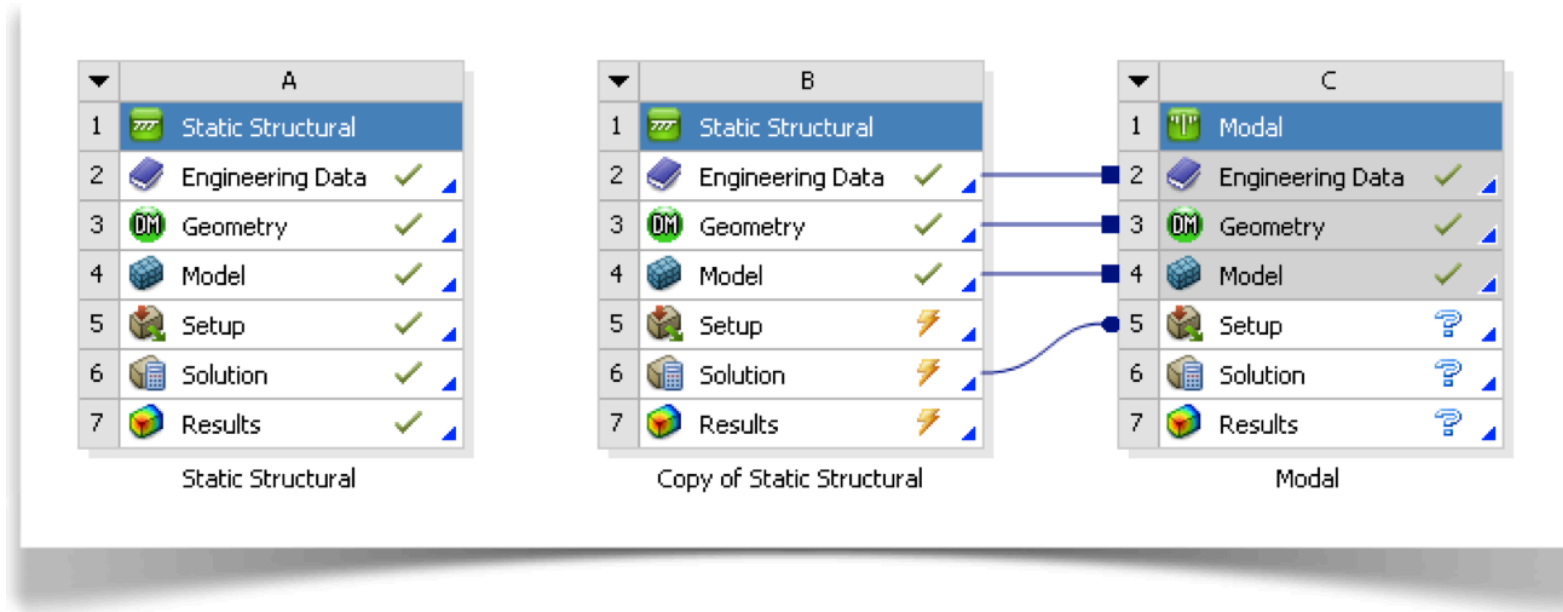
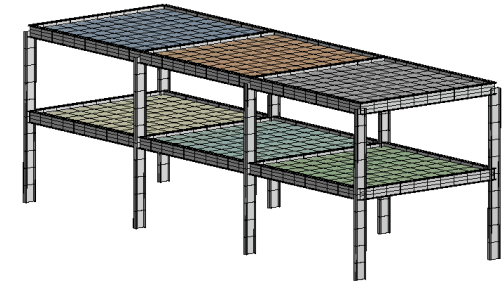
# Techniques/Concepts

- Unprestressed Modal Analysis
- Prestressed Modal Analysis
- Create Mode Shape Results



# Section 11.2

## Two-Story Building



A	
1	Static Structural
2	Engineering Data ✓
3	DM Geometry ✓
4	Model ✓
5	Setup ✓
6	Solution ✓
7	Results ✓

Static Structural

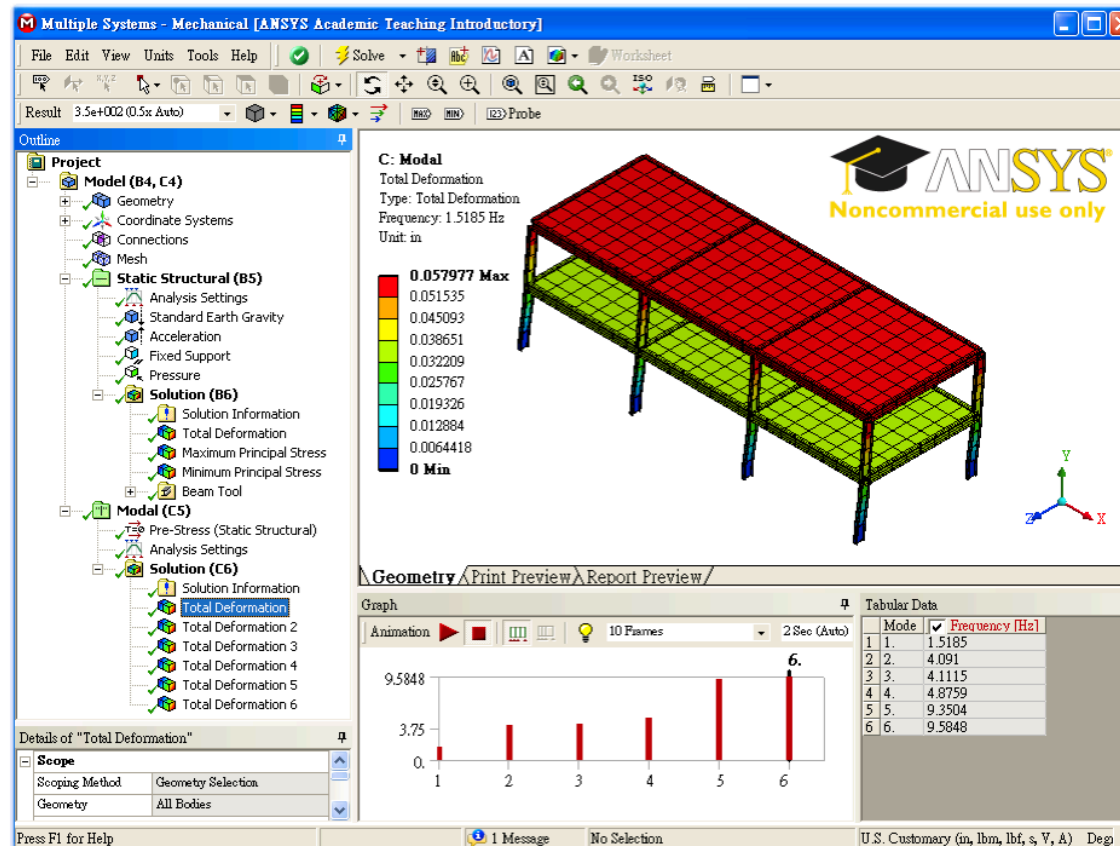
B	
1	Static Structural
2	Engineering Data ✓
3	DM Geometry ✓
4	Model ✓
5	Setup ⚡
6	Solution ⚡
7	Results ⚡

Copy of Static Structural

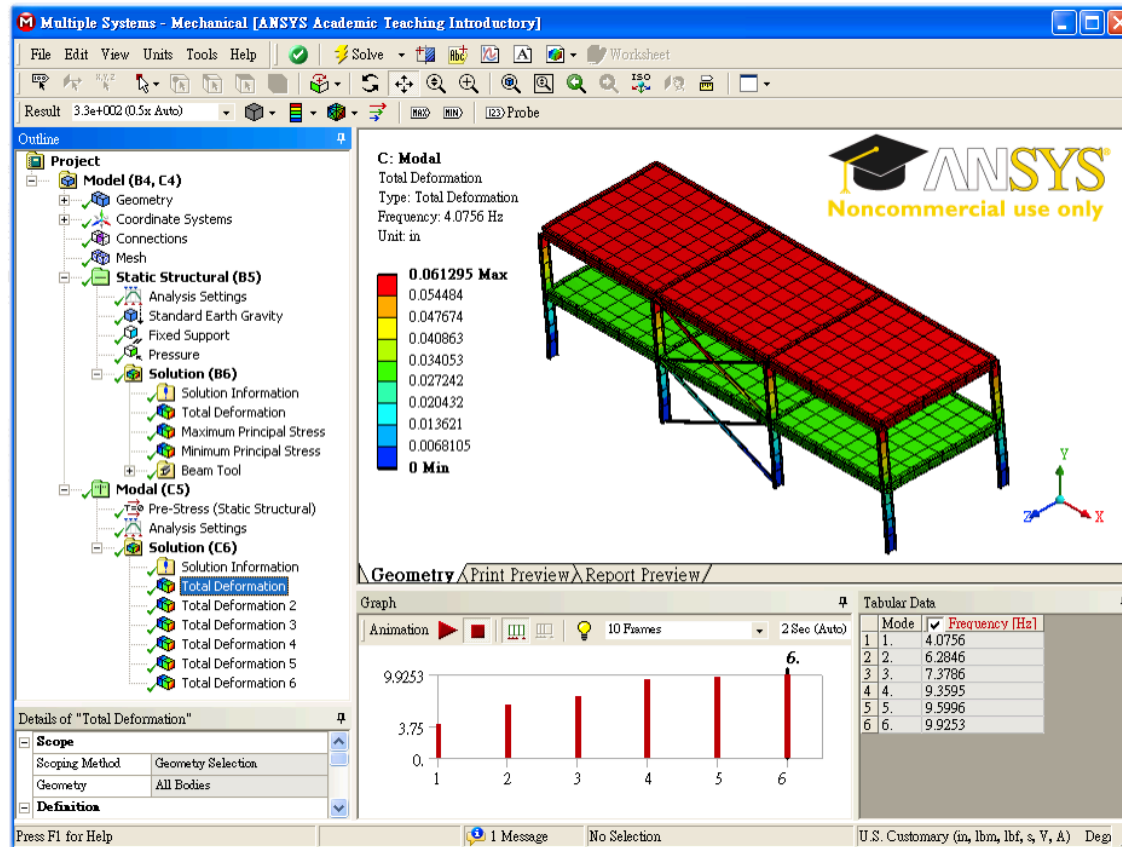
C	
1	Modal
2	Engineering Data ✓
3	DM Geometry ✓
4	Model ✓
5	Setup ?
6	Solution ?
7	Results ?

Modal

# Original Structure...



# After Bracing...



# Section 11.3

## Compact Disk

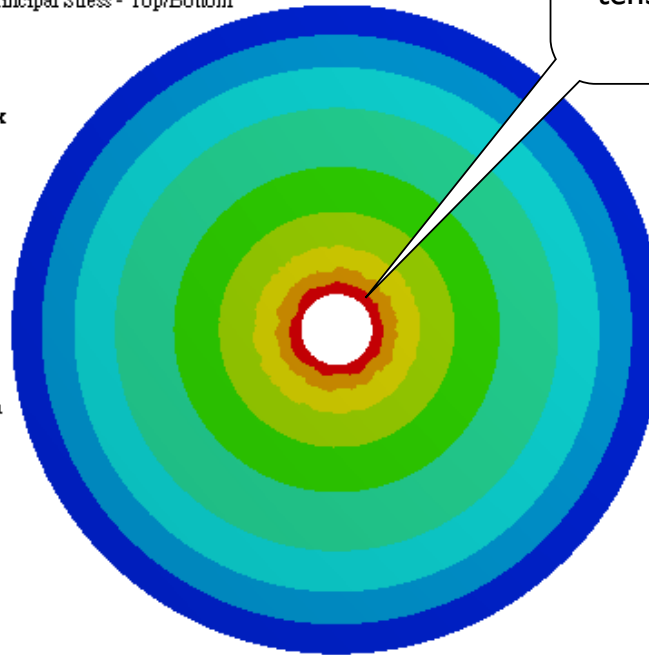
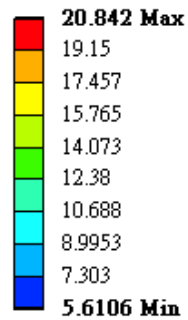


### Problem Description

- The *MythBusters* succeeded in shattering CDs at speeds of 23,000 rpm. We want to find out the maximum stress in the CD due to the centrifugal force to justify if the shattering is indeed due to the high stress.
- We also want to investigate the possibility of resonant vibrations. We will conclude that the CD shattering may be due to vibrations rather than centrifugal stress.

# Stresses Due to Centrifugal Force

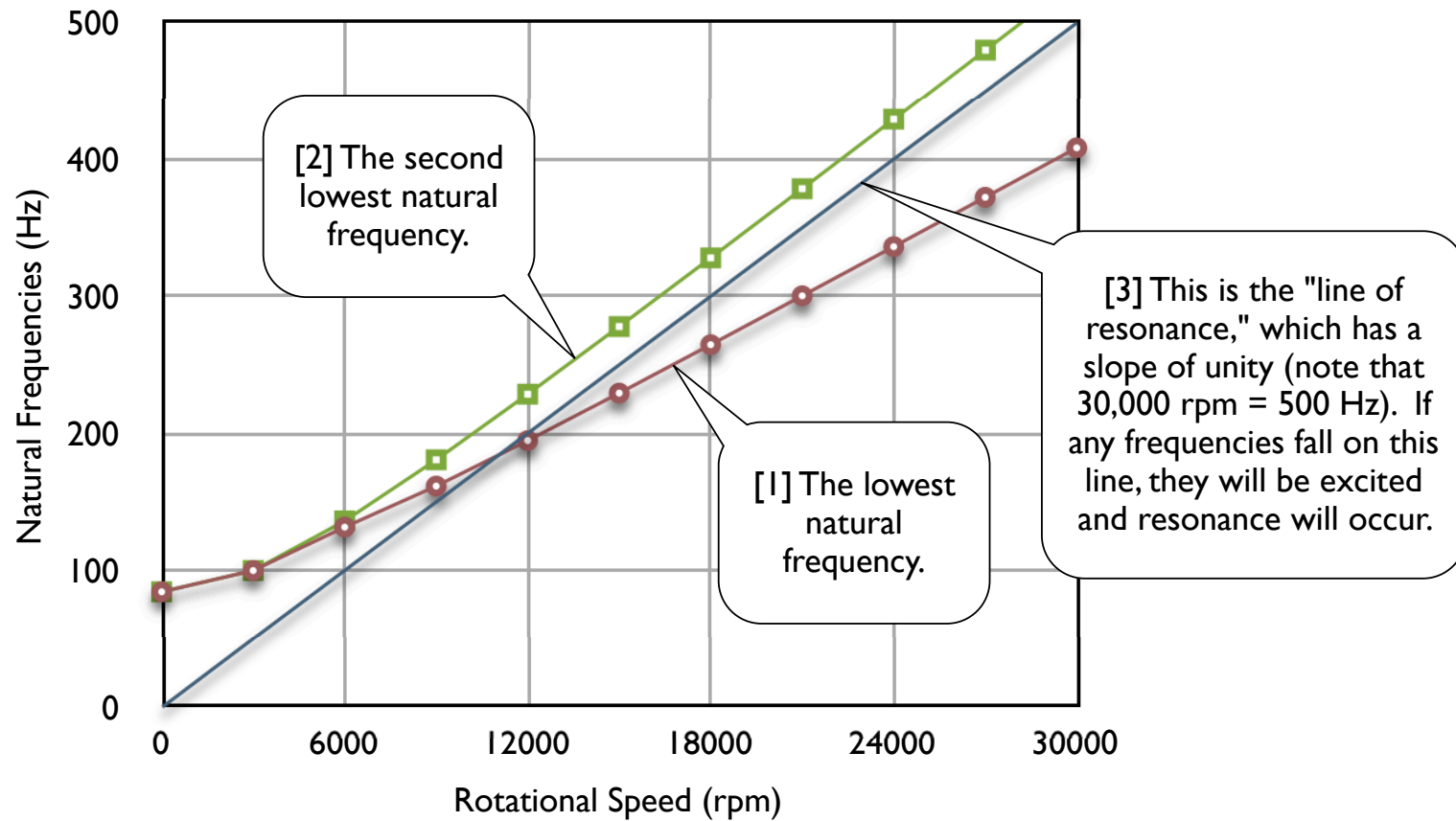
**A: Static Structural**  
Maximum Principal Stress  
Type: Maximum Principal Stress - Top/Bottom  
Unit: MPa  
Time: 1



The maximum tensile stress is 20.8 MPa, far less than the material's tensile strength (65 MPa).



# Natural Frequencies



# Section 11.4

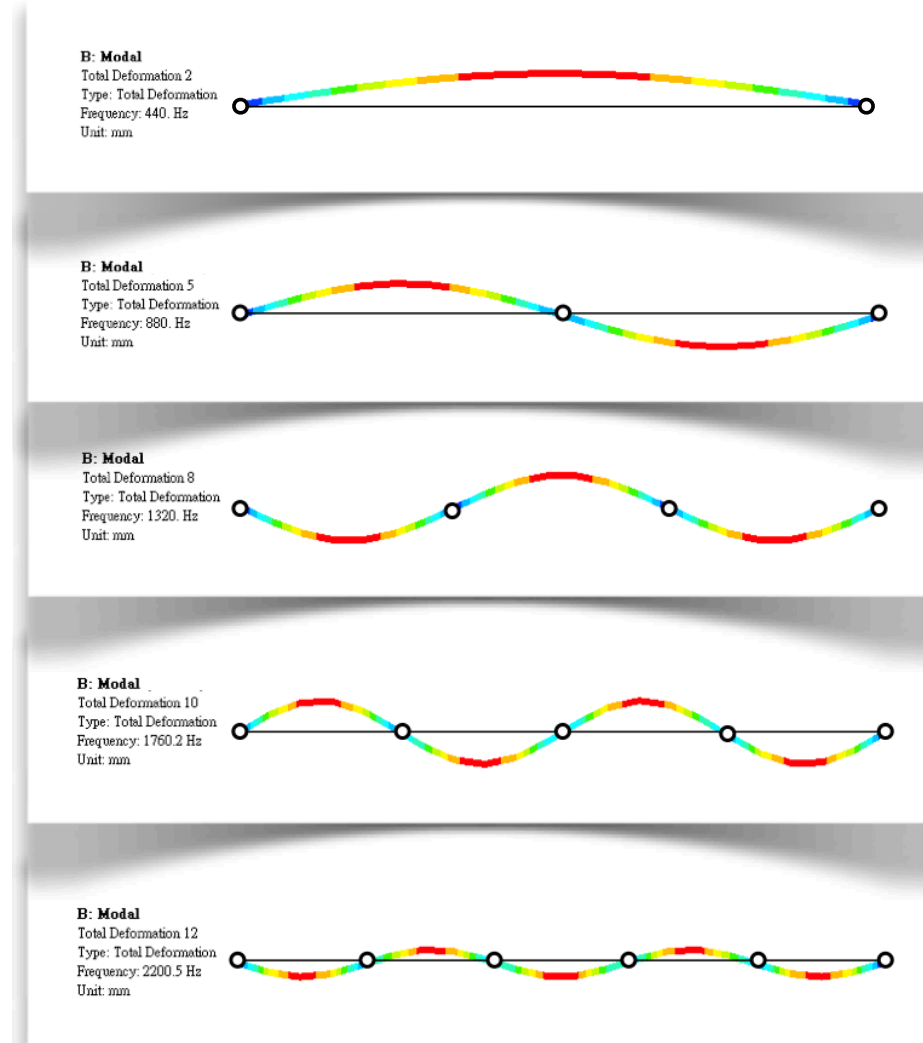
## Guitar String

### Terminologies

- Harmonic Series
- Just Tuning System
- Twelve-Tone Equally Tempered Tuning System
- Beat Frequency

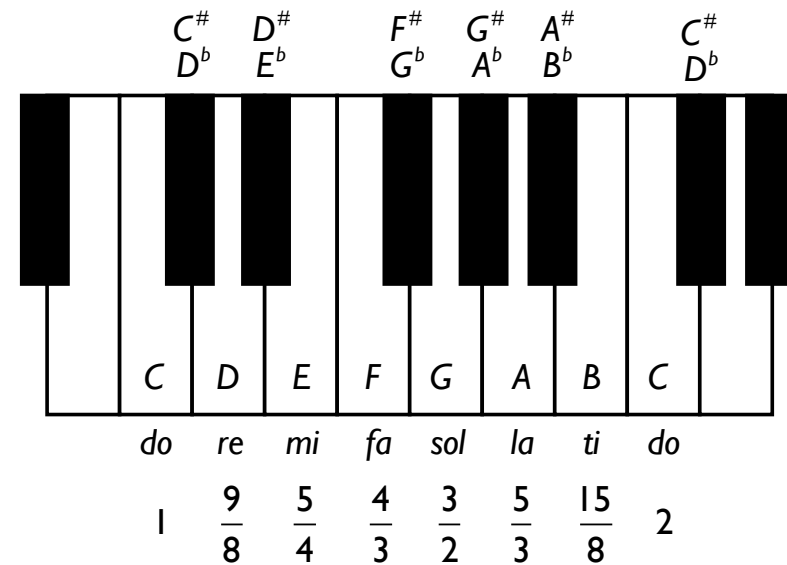
# Harmonic Series

- A *harmonic mode* has a frequency that is an integral multiplication of the fundamental frequency.
- If you pluck a string, you will produce a tone made up of all harmonic modes, the *harmonic mixes* determine the quality of the note.



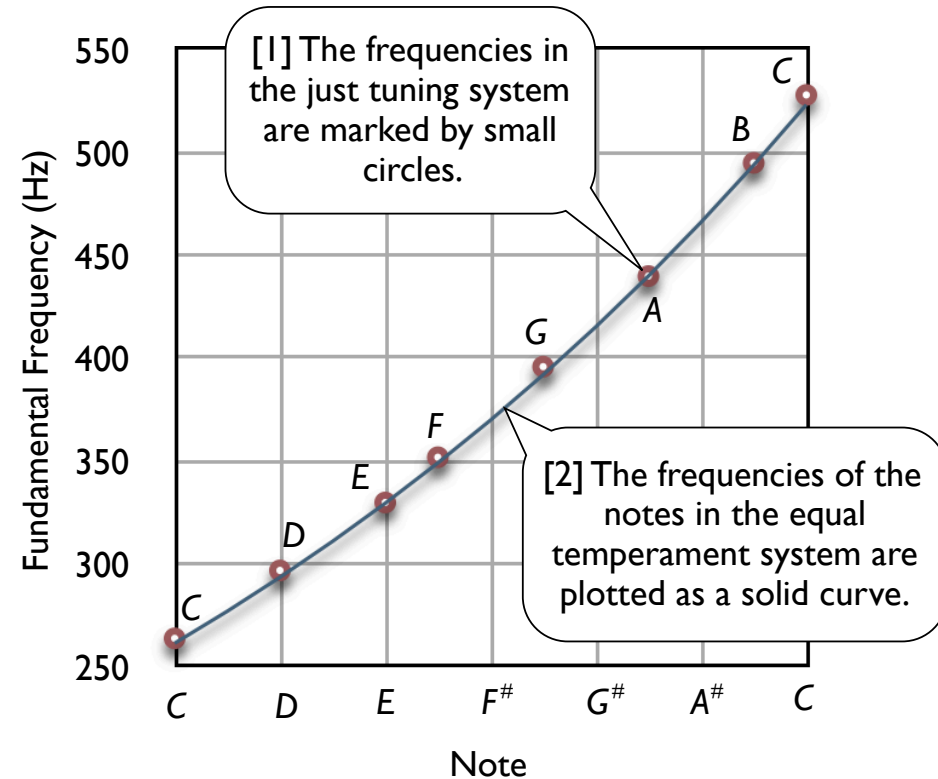
## Just Tuning System

- In the early 20th century, the "just tuning" systems prevail in the music world.
- If we play the notes *do* and *sol* together, the sound is pleasing to our ears, since they have the simplest frequency ratio.
- The major cord C consists of the notes *do, me, sol, do*, the simplest frequency ratios.
- The problem of this system is that it is almost impossible to play in another key.



## Twelve-Tone Equally Tempered Tuning System

- The idea is to compromise on the frequency ratios between the notes, so that they can be played in different keys.
- In this system, an octave is equally divided into 12 tones (including semitones) in logarithmic scale.
- In other words, the adjacent tones have a frequency ratio of  $2^{1/12}$ .



## Beat Frequency

- When two waves of different frequencies are combined, they interfere with each other. The fluctuation in amplitude of the combined wave is called *beats*, and the frequency is called the *beat frequency*.
- The beat frequency is equal to the frequency difference of the two waves.
- When we play *C* and *D* together, the beat frequency is 32.03 Hz ( $293.66 - 261.63$ ), which is a harsh buzz and unpleasant for our ears.
- When we play *C* and *G* together, the beat frequency is 130.37 Hz ( $392.00 - 261.63$ ), which is close to a harmonic frequency of the middle *C* (261.63 Hz).